Inducing a Network Influence Structure from Multiple Diffusion Processes

David Strang[1]
Department of Sociology
Cornell University
Ithaca NY 14853
email : ds20@cornell.edu

January 1996

[1]Research support provided by National Science Foundation grant SES 9213152.

There has been much recent development of models and methods for the analysis of diffusion processes, where adoptions of an 'innovation' by members of a population are treated as interdependent. Perhaps the most lively approach to diffusion has been to treat it as a point-to-point network process, where measured social relations channel influence from prior to potential adopters.

While diffusion research in sociology has traditionally been underpinned by explicit attention to relational networks, the theoretical interest of much contemporary research was re-awakened by Ron Burt's (1987) contrast between influence based on direct relations and influence based on structural equivalence. In a re-analysis of Coleman, Katz, and Menzel's (1966) classic study, *Medical Innovation*, Burt argued that the diffusion of a new prescription drug was channeled by implicit competition among structurally equivalent physicians (physicians with similar patterns of ties to the local medical community as a whole) rather than by direct advising or discussion relations or by aggregated relational properties such as network prominence.

The identification of a diffusion channel that augments or supplants direct ties and which is also grounded in a relational analysis has provided rich fodder for empirical study. A variety of analyses have carefully examined ways to think about how networks mobilize action (for example, see Friedkin 1984, Galaskiewicz and Burt 1991, Mizruchi 1993). For present purposes, the important point is the fruitfulness of explicitly treating diffusion as mobilized by concrete social networks.

Network models of diffusion also articulate well with the development of explicit diffusion models and formal methods. One approach is to employ spatial regression methods that model simultaneous interdependencies in linear outcomes (Doreian, 1981). A second strategy, pursued here, is to elaborate event history methods to include the impact of prior adopters standing in various relations to the focal adopter (Marsden and Podolny, 1990; Strang, 1991; Strang and Tuma, 1993). Event history methodology provides a useful framework for network models of diffusion, both since it is designed to capture the unfolding of a temporal process in the presence of censoring and time varying explanatory factors, and because it permits analysis at the individual rather than the population level. For example, a main strength of the 'heterogeneous diffusion' framework proposed in Strang and Tuma (1993) is its capacity to examine the effects of multiple social networks on diffusion (discussed in terms of 'social proximity' effects).[1]

It is worth noting that alternative notions of what underpins diffusion can are clearly imaginable. For example, one may view diffusion in terms of general social mobilization projects that transcend network relations (Strang and Meyer, 1993). Or social structure may be viewed in terms of distributions of individual characteristics rather than as stable patterns of interaction (Blau, 1977). While plausible, these insights into diffusion are less prominent in sociological theorizing and will be left to one side here.

Network analyses of diffusion are resolutely hypothesis testing in ambition. Analysis begins with arguments about how local social relations drive social influence, and asks whether the timing of adoptions within a population is consistent with that structure. The relevant networks and proximity metrics are defined *a priori*, and estimated models gauge the extent to which diffusion appears to flow along hypothesized lines. Competing hypotheses concern how to translate an incidence matrix into an influence structure (for example, cohesion versus structural equivalence) or what substantive relations are influential.[2]

This paper maintains the social network conception of what drives diffusion, but considers the converse identification problem. I seek to induce a network structure of influence from observations on adoption outcomes. Rather than beginning with measured relations whose apparent mobilization of action is assessed, I generate a social network that reflect apparent interdependencies in outcomes. In a more primitive sense than intended by Strang (1991:324), this effort turns diffusion analysis into 'a search for social structure.'

While Monte Carlo analyses of hypothesis-testing diffusion methods suggest good resolving power in modest samples (Greve, Strang and Tuma 1995), specification of an unrestricted influence network requires more information than is

[1]An alternative approach to diffusion within an event history framework studies parametric hazard models that are motivated by models of interdependent action. See especially Diekmann (1989) and Yamaguchi (1994).

[2]For example, Strang (1990) asks whether decolonization is accelerated by prior decolonization within the empire or within the region.

provided by a single realization of a diffusion process. Given n connected adopters, we require $n^2 - n$ influence parameters for a full influence network. But a diffusion process provides at most $(n^2 + n)/2$ spells to which covariates specifying prior adopter patterns can be attached. And a reasonable estimate of the number of independent bits of information in an event history is a good deal smaller.

Non-parametric induction of a network influence structure thus requires data on multiple diffusion processes (or multiple innovations). It is not clear just how many such processes are needed to permit a substantial analysis. While future investigation may provide a basis for an answer to this question, it seems reasonable to begin by assuming that the number of processes should be at least n. Further, these processes must in some sense be considered as comparable, at least net of modelled heterogeneity. Below, I examine one of the few data sources on diffusion which arguably meets at least the first of these requirements: Jack Walker's (1969) data on the establishment of a variety of policies and programs across the American states.

1 Some Basic Models

I briefly propose a simple hazard model and estimation strategy which may be employed to identify a network influence structure. This strategy is intended to provide a starting point upon which future modelling efforts may improve.

We may consider a quite general hazard formulation:

$$h_{ik}(t) = f(i, k, t, \sum_{j \neq i} \gamma_{ij} Y_{jkt}) \tag{1}$$

where $h(.)$ gives the limiting probability of an adoption at t given that the adoption does not occur before t. Here i refers to an adopter at risk, k to the innovation under examination, and t to time since i became at risk of adopting innovation k. j indexes the other actors in the population, γ_{ij} gives the influence of j on i, and Y_{jkt} is a binary variable that equals 1 if j has adopted innovation k at time t.

The aim is thus to recover the influence matrix $\Gamma = [\gamma_{ij}]$ as a component of the overall function governing the timing of adoptions across adopters i and innovations

k. Difficult problems are posed by the need to specify or control for adopter specific effects, innovation specific effects, and patterns of time dependence within adoption processes. The above formulation simplifies away from some of the complexities by treating adopter effects as temporally stable, but real problems remain.

A useful starting point is to condition on the presumably large variability across adopters by estimating all parameters within each individual. Rather than jointly model the hazards of multiple population members, I thus compare the (time-varying) rates at which each i adopts a series of innovations, modelled as a function of the influence vector $\mathbf{Y}_{jkt}$. To the extent that i's intrinsic propensity to adopt is stable over time and homogeneous across innovations, adopter specific variation in the hazard drops out of estimation. Further, since there is no overlap in the parameter vectors γ_{ij} of different i's, no precision is lost by analyzing each adopter independently.

To decrease heterogeneity due to the innovation, I standardize the overall speeds of the k adoption processes. For each innovation I calculate the hazard under the time and adopter homogeneous null model $h_k(t) = \alpha_k$. The time scale of each process is then expanded or compressed such that all times t_k become $t_k\alpha_k$. This permits the analysis to omit calculating a parameter for each innovation k, which given the conditioning operation on the adopter would presumably necessitate a two-stage estimation process.

Finally, as an first cut I assume that there is no time dependence in the adoption processes other than that produced by the influence of other adopters. This is clearly a strong assumption, and one which may be remedied in further analysis embedding estimation within a partial likelihood framework. Alternatively, one might directly model the impact of changing adopter and environmental characteristics.

This estimation strategy thus leads to I analyses of models taking the form:

$$h_{ik}(t_{h_k}) = \exp(\alpha_i + \sum_{j\neq i} \gamma_{ij} Y_{jkt}). \tag{2}$$

The individual-specific term α_i is estimated as a constant term in each of the I analyses. Each analysis estimates the influence of the set of population members j on a particular individual across the full set of K innovations.

Models are estimated within a maximum likelihood framework. I make use of SAS-IML software to conduct the substantial matrix operations required to work with vectors like $\gamma_{ij}\mathbf{Y}_{jkt}$ across multiple processes. Parameter coefficients are calculated via NLP/NRA, a Newton-Raphson nonlinear optimizer provided in SAS.

In analyses reported below, states are treated as at risk of adoption of an innovation from the time of the first historical adoption of that innovation or the date when they were organized, whichever occurred later. The small number of adoptions which occurred to territories prior to their elevation to statehood are not modelled but are treated as influential.

2 Data

The above modelling strategy is applied to data on public policy innovation presented by Jack Walker. Walker's (1969) paper, 'The Diffusion of Innovations among the American States,' is perhaps the classic study of policy diffusion within political science. Walker identified the years when 48 states enacted 88 different programs (Alaska and Hawaii were omitted from Walker's data collection). Of these 88 programs, data on 85 are available here for re-analysis. These programs embrace a variety of issue areas, such as welfare, civil rights, administrative organization, and tax reform, and cover a time span of some two centuries. A list of the 85 programs under study is provided in the appendix.

Walker developed an innovativeness score based on the speed with which states adopted programs, and related it to the socio-economic and political characteristics of the states. Most dramatically, however, Walker proposed that policies diffuse via a distinct network structure of learning and emulation. Stimulated by qualitative impressions of inter-state imitation and influence, Walker pictured the policy network in the form of a regionally organized tree. In his words,

> 'At the top of the tree would be a set of pioneering states which would be linked together in a national system of emulation and competition. The rest of the states would be sorted out along the branches of the tree according to the pioneer, or set of pioneers, from which they take

> their principal cues. States like New York, Massachusetts, California, and Michigan should be seen as regional pace setters, each of which has a group of followers, usually within their own region of the country, that tend to adopt programs only after the pioneers have led the way.' [1969: 893]

Partial confirmation of the operation of a regional network structure was suggested by a factor analysis of resemblances in the timing of policy adoptions across states. Walker computed time to adoption of each state for each policy as the percentage of time between the first and last adoption at which the state adopted.[3] He then correlated the adoption scores of each pair of states, and studied the resulting state-by-state matrix. Seven factors were recovered, several of which bore strong regional markings. In particular, all of the Southern states loaded highly on the first factor (along with six non-Southern states), and the second factor was composed of all five New England states plus New York and Pennsylvania.

This correlation-based factor analytic approach, while suggestive, has several obvious limitations.[4] Perhaps most importantly, the use of correlations treats diffusion influences as definitionally symmetric. Consider the case where Delaware frequently adopts legislation shortly after Maryland does, while early adoptions by Delaware are not systematically accompanied by subsequent early adoptions by Maryland. While this would seem strong evidence that Maryland influences Delaware but not vice versa, any correlation-based method will treat the 'influence' of Delaware on Maryland as identical to that of Maryland on Delaware.

In addition, a correlational metric simply indexes the linearity of the relationship between the policy adoptions of a pair of states. But evidence of strong influence requires proximity in time as well as appropriate time ordering. The fact that one state typically takes twice as long to adopt legislation as another is not suggestive of influence.

A factor analytic strategy raises further concerns by presuming that influence

[3]For example, if the first state adopted Fair Housing Legislation in 1952 and the last adopted it in 1964, then a state adopting in 1955 would receive a score of 0.25.

[4]I leave aside the loss of information in such procedures as the computation of innovation scores to focus on the more substantive implications of Walker's approach.

takes the form of some sort of clique structure. While states can potentially load on more than one factor, the notion that factors can represent patterns of influence limits the range of identifiable network structures. At least as conventionally deployed, it is not clear that a factor analysis can properly recover a simple binary tree. Further, the fact that a factor solution can be rotated without loss of information leads whatever solution the analyst identifies to be suspect.

3 Results

Following the strategy outlined above, I modelled each state's adoption of the Walker's 85 programs separately. To give some flavor of the results of these analysis, Table 1 gives parameter estimates for Conneticut.

Table 1 about here.

Other than the constant given in the first row, each covariate in Table 1 provides the effect of some other state's adoption of an innovation k on Conneticut's hazard of adopting innovation k. For example, the third row indicates that a prior adoption by Massachusetts increases the rate at which Conneticut adopts policies almost fourfold [$\exp(1.33) = 3.78$]. This effect is about three times larger than its standard error and significant at any conventional significance level. Note that prior adoption by some other states (such as Delaware and Ohio) tends to depress the speed with which Conneticut establishes programs.

In general, the case of Conneticut points to some fairly strong regional linkages. Conneticut's neighbors (Massachusetts, Rhode Island, and New York) make up three of the six states that accelerate Conneticut's policy adoptions, and a fourth (Pennsylvania) is also geographically proximate. The demographic and socio-economic profiles of these states are presumably quite similar to Conneticut. Three of these four regional states are considerably larger than Conneticut, and perhaps highly visible to it as a result; it is of interest that the only state that both influences and is influenced by Conneticut is Rhode Island. While the other influences on

Conneticut are not easily interpreted (positive influences of Mississippi and Arizona, negative influence of Ohio), the overall picture is highly compatible with regionally structured influence.

It would be tedious to examine each of the 48 states in similar fashion. Instead, Table 2 simplifies the picture by giving the main influencers for each state, defined as the states whose influence coefficients are statistically significant at the .01 level.

Table 2 about here.

The table is organized in terms of regions, since geographically mediated linkages are easily picked up by eye. (Future investigation will examine the induced influence network from other points of view.) Influencers whose regional grouping is the same as the adopter are shown in bold; influencers that are geographically contiguous with the adopter are underlined.

The first portion of the chart shows influences on New England and Middle Atlantic states. As the above discussion of Conneticut suggests, much of the inter-state influence appears to be regional. 13 of the 23 positive influences occur within the 10 states making up this region, a much larger proportion than the ratio of 0.03 one would expect by chance $[(10^2 - 10)/(48^2 - 48) = 90/2256]$. A considerable number of these influencers are in fact contiguous with the adopters of interest (9 of 13), and there is one additional case of an influencer that is in a different region but is in fact a neighbor (Maryland's influence on Delaware). Finally, no negative influences are found within the New England and Middle Atlantic region.

Before moving on, it is worth noting that there is some hint that inter-regional influence is stronger for the smaller states in the region than the larger ones. Conneticut, Maine, Rhode Island, and Delaware seem particularly strongly linked to other New England and Middle Atlantic states. By contrast, the larger states in the region (Massachusetts, New York, Pennsylvania) show fewer regional influences and a higher number of effects from states outside the region.

The second, third, and fourth listings in Table 2 show influence patterns for Midwestern, Southern, and Mountain and Pacific states respectively. All of these groupings show a bias towards inter-regional influence. For example, in all three groups the ratio of positive influences within the region to all positive influences is much greater than would be expected by chance. And only in the Southern region do appreciable numbers of negative within-region influences appear.

However, it is important to note that none of these three other groupings show the degree of cohesion displayed by the New England and Middle Atlantic states. The Midwestern and the Mountain and Pacific states show clear though more modest signs of inter-regional influence. These regions also support the notion that the most prominent states within a region may be least connected to their neighbors. Illinois, Michigan, and Texas are not influenced by any other states; California is peculiarly linked to Missouri, Arizona, Maryland, and South Carolina.

The least cohesive grouping is formed by the Southern states, where only five of fifteen states are positively influenced by others within the region. While part of this result may be attributed to the inclusion of Western-oriented states like Texas and Oklahoma in this group, the absence of strong connections within the deep South remains surprising.

This exploratory analysis of inter-state influence thus provides some support for Walker's (1969) image of a 'national' league of major states and regionally clustered 'bush' leagues. Walker's tight cluster of New England states is strongly mirrored in the influence structure recovered in the above analysis. The ambiguous position of presumably salient 'big league' states like California is also suggested in both studies.

The most concrete difference between the two sets of results is the absence of strong effects within the Southern states here, as opposed to Walker's extraction of a 'Southern factor.' But more generally, the analysis presented here suggests greater complexity and less attachment to a fundamentally regional structure than Walker proposes. While strong regional effects are clearly one of the operative elements in the influence network, there is much heterogeneity in the degree to which different states are linked to their neighbors. Only 25 of the 48 states show a strong positive influence of another state in their region. Further work may probe the sources of greater versus less regionalism in influence across adopters, as well as search for bases of influence other than geographic proximity.

4 Discussion

This paper suggests an event history strategy for inducing a network structure of influence from data on multiple diffusion processes. While the strategy utilised here is a simple one that surely invites further development, it has the virtue of permitting any sort of network structure emerge from empirical data. Relative to the factor analytic strategy proposed by Walker, for example, it does not imply that influence is symmetric or that adopters can be partitioned into cliques. Instead, a full influence matrix is generated which can then itself be probed.

The application of this method to Walker's public policy data suggests several patterns in the way American states may influence each other. There are clear indications of within-regional influence, particularly in the Northeast and least strongly in the South. Smaller states seem particularly affected by their neighbors, while the larger states are less tied to other states within the region (and perhaps outside as well). And some regions appear more cohesive than others.

While Walker's data on state policies provides considerable ammunition for an empirical analysis, the way it exposes the simplicity of the model examined here should also be noted. The policy decisions under study span almost two centuries and a wide variety of types of issues, from highly contested and differentially salient civil rights legislation to routine administrative practices like the establishment of a fishing board. While it can be argued that all kinds of policies in all eras tend to diffuse, it is unclear whether it is reasonable to assume that the structure of influence is stable over time and across policy issues. A remedy for the heroic assumptions of stability across processes and historical time is needed to move beyond the first cut provided in this paper.

In addition to refining the strategy for inducing a network structure from data on multiple diffusion processes, there is much scope for expanded analysis of the recovered influence matrix. The discussion above has focused on whether we should think of inter-state influence as importantly regional, and suggests some interesting complexities in the degree to which different states are tied to their neighbors. Further work can examine other bases of influence: immediate candidates for investigation might be hierarchies of size and wealth, similarity in socio-economic conditions, and common political and cultural traditions. And we can inquire whether inter-state influence appears to be driven by direct interactions (as measured in trade networks or proxied by region), by equivalent location in a national status and economic system, or by other ways of thinking about how social units are connected to each other. Such analyses can add powerfully to hypothesis testing approaches to diffusion by making influence networks observed as well as theorized structures, and dependent as well as independent variables.

References

Blau, Peter M. (1977). *Inequality and Heterogeneity.* Free Press, New York.

Burt, Ronald S. (1987). "Social contagion and innovation: cohesion versus structural equivalence". *American Journal of Sociology*, 92:1287–1335.

Coleman, James S., Katz, Elihu, and Menzel, Herbert (1966). *Medical Innovation.* Bobbs-Merrill, New York.

Diekmann, Andreas (1989a). "Diffusion and survival models for the process of entry into marriage". *Journal of Mathematical Sociology*, 14:31–44.

Doreian, Patrick (1981). "Estimating linear models in spatially distributed data". In *Sociological Methodology*, edited by S. Leinhardt, chapter 10, pages 359–388, Jossey-Bass, San Francisco.

Friedkin, Noah E. (1984). "Structural cohesion and equivalence explanations of social homogeneity". *Sociological Methods and Research*, 12:235–61.

Galaskiewicz, Joseph and Burt, Ronald S. (1991). "Interorganizational contagion in corporate philanthropy". *Administrative Science Quarterly*, 36:88–105.

Marsden, Peter V. and Podolny, Joel (1990). "Dynamic analysis of network diffusion processes". In *Social Networks through Time*, edited by H. Flap and J. Weesie, ISOR, Utrecht.

Mizruchi, Mark S. (1993). "Cohesion, equivalence, and similarity of behavior: a theoretical and emp assessment". *Social Networks*, 15:275–307.

Strang, David (1990). "From dependency to sovereignty: an event history analysis of decolonization". *American Sociological Review*, 55:846–860.

Strang, David (1991). "Adding social structure to diffusion models: an event history framework". *Sociological Methods and Research*, 19:324–353.

Strang, David and Meyer, John W. (1993). "Institutional conditions for diffusion". *Theory and Society*, 22:487–512.

Strang, David and Tuma, Nancy B. (1993). "Spatial and temporal heterogeneity in diffusion". *American Journal of Sociology*, 99:614–39.

Walker, Jack (1969). "The diffusion of innovation among the American states". *American Political Science Review*, 63:880–899.

Yamaguchi, Kazuo (1994). "Some accelerated failure-timw regression models derived from diffusion process models: an application to a network diffusion analysis". In *Sociological Methodology 1994*, edited by P. Marsden, pages 267–300, Blackwell.